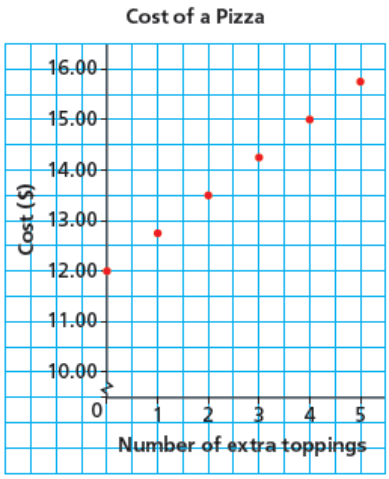
5.6 Properties of Linear Relations

LESSON FOCUS Identify and represent linear relations in different ways.

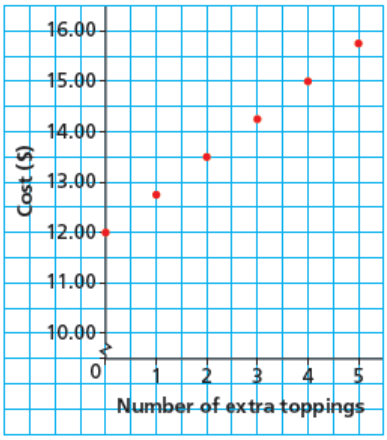
Make Connections

The table of values and graph show the cost of a pizza with up to 5 extra toppings.

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75



Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75



What patterns do you see in the table?

Write a rule for the pattern that relates the cost of a pizza to the number of its toppings.

How are the patterns in the table shown in the graph?

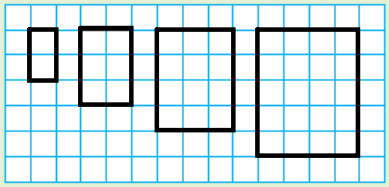
How can you tell from the table that the graph represents a linear relation?

TRY THIS

Work with a partner.

You will need 1-cm grid paper.

Use this pattern of rectangles.
This pattern continues.



A. Draw the next two rectangles in the pattern.
Copy and complete each table of values for the 6 rectangles.

Width of Rectangle (cm)	Area (cm^2)
1	
2	

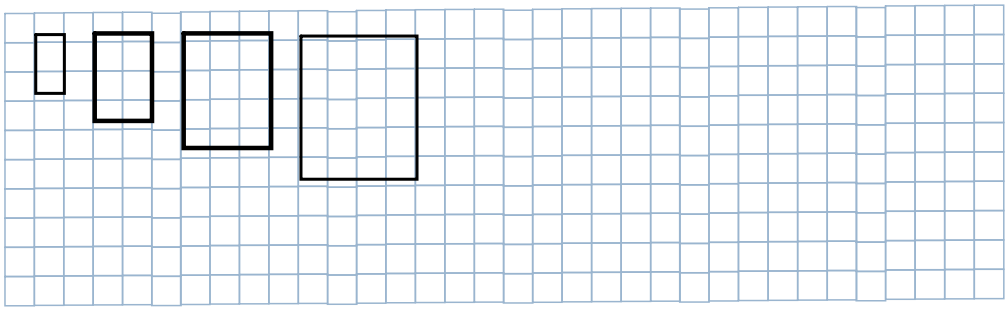
Width of Rectangle (cm)	Perimeter (cm)
1	
2	

TRY THIS (continued)

B. Which table of values represents a linear relation? How can you tell?

C. Graph the data in each table of values.
Does each graph represent a linear relation?
How do you know?

Draw the next two rectangles in the pattern.



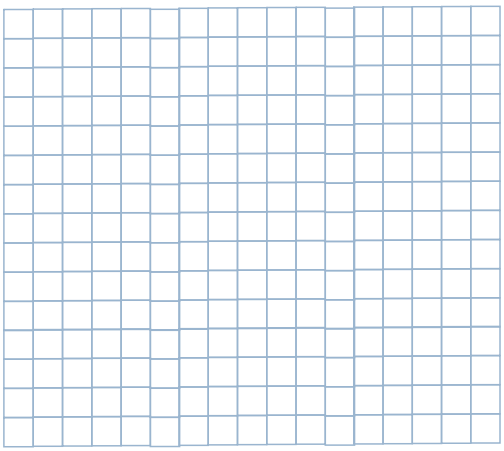
Copy and complete each table of values for the 6 rectangles.

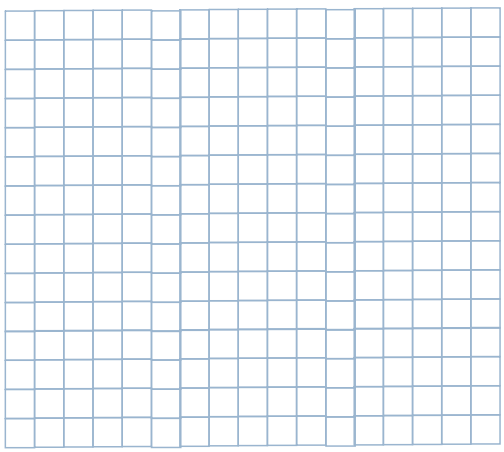
Width of Rectangle (cm)	Area (cm^2)

Width of Rectangle (cm)	Perimeter (cm)

Which table of values represents a linear relation? How can you tell?

Graph the data in each table of values.
Does each graph represent a linear relation? How do you know?





The cost for a car rental is $60, plus $20 for every 100 km driven.
The independent variable is the ? and the dependent variable is ?

We can identify that this is a linear relation in different ways.

#1 a table of values

Distance (km)	Cost ($)
0	60
100	80
200	100
300	120
400	140

*

5.6 Properties of Linear Relations

Tuesday, November 29th

- Review notes from section 5.6
- Look over a few examples (copy into notes)
- Classwork/Homework

Please note: Extra help will now be offered on Mondays and Thursdays (11:50 - 12:20)

We can identify that this is a linear relation in different ways.

#1 a table of values

# of Km (x)	Cost ($) (y)
0	60
100	80
200	100
300	120
400	140

+100, +100, +100, +100 (x-values)

+20, +20, +20, +20 (y-values)

$60 to rent a car and $20 / 100 Km

#2 a set of ordered pairs

The difference in each x-value must be equal.
The difference in each y-value must also be equal.

+100 +100 +100 +100

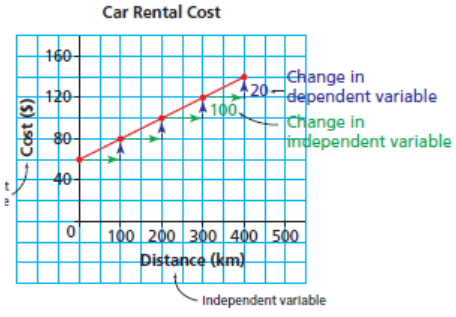
{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)}

+20 +20 +20 +20

#3 a graph

Dependent variable

A linear relation's graph will be one straight line in any direction.



5.6 Properties of Linear Relations

We can use each representation to calculate the rate of change. (aka "slope")

The rate of change can be expressed as a fraction:

$$\frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{\$20}{100 \text{ km}}$$

$$= \$0.20/\text{km}$$

The rate of change is \$0.20/km; that is, for each additional 1 km driven, the rental cost increases by 20¢. The rate of change is constant for a linear relation.

We can determine the rate of change from the equation that represents the linear function.

Let the cost be C dollars and the distance driven be d kilometres.

An equation for this linear function is:

$$C = 0.20d + 60$$

Dependent variable

Example 1 Determining whether a Table of Values Represents a Linear Relation

Which table of values represents a linear relation? Justify the answer.

a) The relation between temperature in degrees Celsius, C, and temperature in degrees Fahrenheit, F

C	F
0	32
5	41
10	50
15	59
20	68

b) The relation between the current, I amps, and power, P watts, in an electrical circuit

I	P
0	0
5	75
10	300
15	675
20	1200

CHECK YOUR UNDERSTANDING

SOLUTION

rate of change: $\frac{9}{5} = 1.8$

When an equation is written using the variables x and y, x represents the independent variable and y represents the dependent variable.

Example 2 Determining whether an Equation Represents a Linear Relation

a) Graph each equation.

i) $y = -3x + 25$

ii) $y = 2x^2 + 5$

iii) $y = 5$

iv) $x = 1$

b) Which equations in part a represent linear relations? How do you know?

CHECK YOUR UNDERSTANDING

SOLUTION

Example 2 Determining whether an Equation Represents a Linear Relation

a) Graph each equation.

Creating a table of values

i) $y = -3(x) + 25$

x	y	
0	25	-3
1	22	-3
2	19	-3
3	16	-3
4	13	

ii) $y = 2(x)^2 + 5$

x	y	
-2	13	-6
-1	7	-2
0	5	+2
1	7	+6
2	13	

$2(2)^2+5$

0

2 X

iii) $y = 5$

iv) $x = 1$

b) Which equations in part a represent linear relations? How do you know?

Example 3 Identifying a Linear Relation

Which relation is linear? Justify the answer.

a) A new car is purchased for $24 000. Every year, the value of the car decreases by 15%. The value is related to time.

x (time)	y (cost/value)	
0	24 000	-3600
1	20 400	-3060
2	17 340	
3	14 739	
4		

85%

$\frac{85}{100}$

$= 0.85$

CHECK YOUR UNDERSTANDING

b) For a service call, an electrician charges a $75 flat rate, plus $50 for each hour he works. The total cost for service is related to time.

SOLUTION

x (time)	y (cost)	
0	75	+50
1	125	+50
2	175	+50
3	225	+50
4	275	

5.6 Properties of Linear Relations

Pg. 308-309

#3-6

Wednesday, November 30th

- Warm-up #8
- Check and go over homework Pg.308 #3-6
- Review an example on finding rate of change
- Classwork/Homework

Please note: Extra help will now be offered on Mondays and Thursdays (11:50 - 12:20)

Warm-up #8

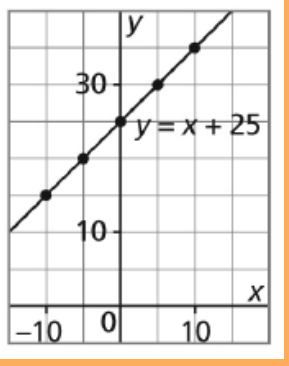
Nov. 30

#1 Are the following linear relations? Show or explain why.

a)

t	n
0	1
20	2
40	4
60	8
80	16
100	32

b)



#2 a. Create a table of values for each of the following.
b. Tell me whether each is a linear relation or not?

a) $y = x^2 + 25$

b) $y = x + 25$

Please open your notebook to the answers to the following questions:

Pg. 308-309 #3-6

3. Which tables of values represent linear relations? Explain your answers.

a)

Time (min)	Distance (m)
0	10
2	50
4	90
6	130

b)

Time (s)	Speed (m/s)
0	10
1	20
2	40
3	80

c)

Speed (m/s)	Time (s)
15	7.5
10	5
5	2.5
0	0

d)

Distance (m)	Speed (m/s)
4	2
16	4
1	1
9	3

3. a) Linear relation **b)** Not a linear relation
c) Linear relation **d)** Not a linear relation

4. Which sets of ordered pairs represent linear relations? Explain your answers.

a) $\{(3, 11), (5, 9), (7, 7), (9, 5)\}$

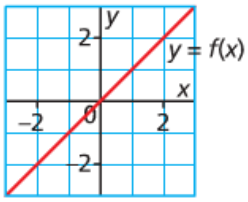
b) $\{(-2, 3), (0, 1), (2, -3), (4, -7)\}$

c) $\{(1, 1), (1, 3), (2, 1), (2, 3)\}$

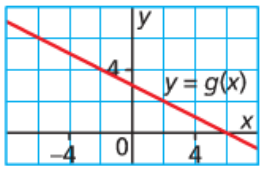
4. a) Linear relation **b)** Not a linear relation
c) Not a linear relation

5. Which graphs represent linear relations? How do you know?

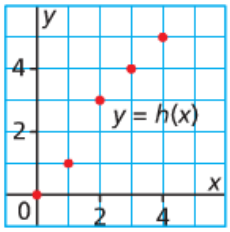
a)



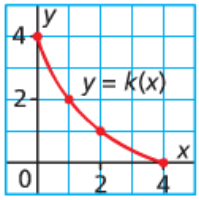
b)



c)



d)



5. a) Linear relation **b)** Linear relation
c) Not a linear relation **d)** Not a linear relation

6. a) Create a table of values when necessary, then graph each relation.

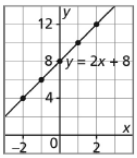
i) $y = 2x + 8$ **ii)** $y = 0.5x + 12$
iii) $y = x^2 + 8$ **iv)** $y = 2x$
v) $x = 7$ **vi)** $x + y = 6$

b) Which equations in part a represent linear relations? How do you know?

6. a) Tables of values may vary. For example:

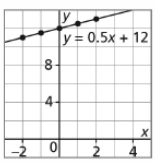
i)

x	y
–2	4
–1	6
0	8
1	10
2	12



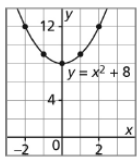
ii)

x	y
–2	11
–1	11.5
0	12
1	12.5
2	13



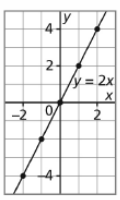
iii)

x	y
–2	12
–1	9
0	8
1	9
2	12

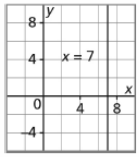


iv)

x	y
–2	–4
–1	–2
0	0
1	2
2	4

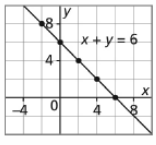


v)



vi)

x	y
–2	8
0	6
2	4
4	2
6	0

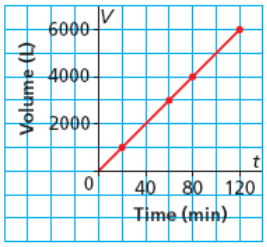


b) The relations in part a, i, ii, iv, v, and vi are straight lines, so they are linear relations.

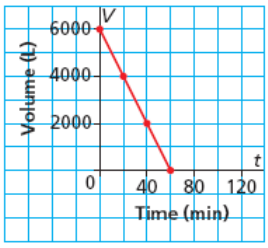
Example 4 Determining the Rate of Change of a Linear Relation from Its Graph

A water tank on a farm near Swift Current, Saskatchewan, holds 6000 L.
Graph A represents the tank being filled at a constant rate.
Graph B represents the tank being emptied at a constant rate.

Graph A
Filling a Water Tank



Graph B
Emptying a Water Tank



CHECK YOUR UNDERSTANDING

a) Identify the independent and dependent variables.
b) Determine the rate of change of each relation, then describe what it represents.

5.6 Properties of Linear Relations

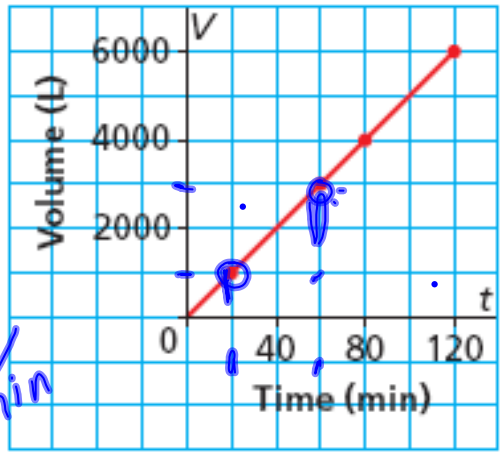
SOLUTION

Rate of change

$$\frac{\text{change of dep. (y)}}{\text{change of ind. (x)}}$$

$$\frac{2000}{40} = 50 \text{ L/min}$$

Graph A
Filling a Water Tank

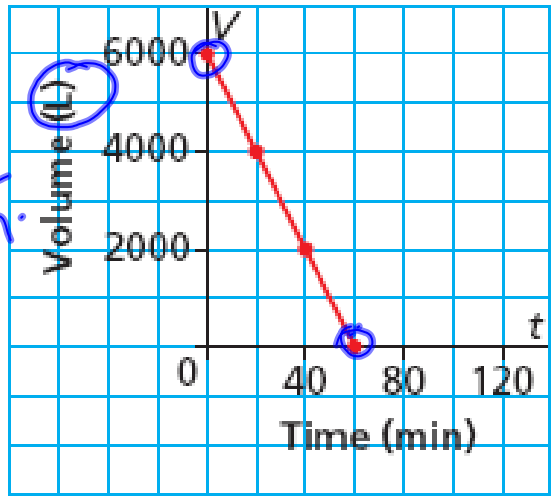


$\frac{\Delta y}{\Delta x}$

$\frac{6000}{60} = 100\ \frac{L}{min}$

Graph B

Emptying a Water Tank



Classwork/Homework

Page 308-310 #7-10,12,14

Thursday, December 1st

- Finish Pg.309-311, show me your completed work. #7-10,12,14
- Begin Section 5.7 (notes/examples)
- Classwork/Homework

Please note: Extra help will now be offered on Mondays and Thursdays (11:50 - 12:20)

Classwork/Homework

Page 308-310 #7-10,12,14

7. For each relation below:

i) Identify the dependent and independent variables.

ii) Use the table of values to determine whether the relation is linear.

iii) If the relation is linear, determine its rate of change.

a) The distance required for a car to come to a complete stop after its brakes are applied is the *braking distance*. The braking distance, d metres, is related to the speed of the car, s kilometres per hour, when the brakes are first applied.

s (km/h)	d (m)
50	13
60	20
70	27
80	35

b) The altitude of a plane, a metres, is related to the time, t minutes, that has elapsed since it started its descent.

t (min)	a (m)
0	12 000
2	11 600
4	11 200
6	10 800
8	10 400

7. a) i) Independent variable: s; dependent variable: d

ii) Not linear

b) i) Independent variable: t; dependent variable: a

ii) Linear

iii) –200 m/min

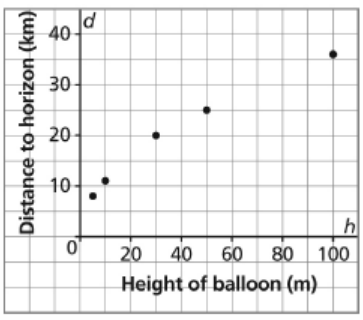
8. In a hot-air balloon, a chart shows how the distance to the horizon, d kilometres, is related to the height of the balloon, h metres.

a) Graph these data.

b) Is the relation linear? What strategy did you use?

h (m)	d (km)
5	8
10	11
30	20
50	25
100	36

8. a)



b) The relation is not linear because the points on the graph do not lie on a straight line.

9. Earth rotates through approximately 360° every 24 h. The set of ordered pairs below describes the rotation. The first coordinate is the time in hours, and the second coordinate is the approximate angle of rotation in degrees. Describe two strategies you could use to determine if this relation is linear.

{(0, 0), (6, 90), (12, 180), (18, 270), (24, 360)}

9. Answers may vary. For example:
I could examine the change in the first and second coordinates. If both changes are constant, the relation is linear.
I could also graph the ordered pairs. If the points lie on a straight line, the relation is linear.

10. Sophie and 4 of her friends plan a trip to the Edmonton Chante for one night. The hotel room is $95 for the first 2 people, plus $10 for each additional person in the room. The total cost is related to the number of people. Is the relation linear? How do you know?

10. Yes

12. The cost, C dollars, to rent a hall for a banquet is given by the equation $C = 550 + 15n$, where n represents the number of people attending the banquet.

a) Explain why the equation represents a linear relation.

b) State the rate of change. What does it represent?

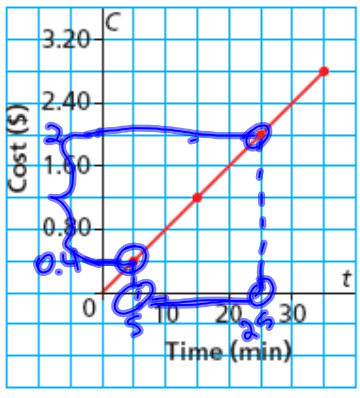
12. a) Answers may vary. For example: The equation relates the dependent variable, C, to the rate of change, 15, times the independent variable, n, plus a constant, 550.

b) 15; cost per guest

14. This graph represents Jerome's long distance phone call to his pen pal in Nunavut. Jerome is charged a constant rate.

a) Identify the dependent and independent variables.

b) Determine the rate of change, then describe what it represents.



dep / ind.

$\frac{\Delta y}{\Delta x} = \frac{1.6}{20}$

$0.08/min.